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Fuzzy Rule-Based Investigation of System Reliability with Reliability Block Diagram Method

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Abstract — **This study analyzes the dependability of a simple network architecture using a fuzzy method that is similar to a stochastic one. This concept is taken from the Fuzzy Fault Tree Analysis (FFTA), which considers the OR and AND gate connections between sets and the maximum and minimum of the membership function inside each set. A Fuzzy Reliability Block Diagram (FRBD) has a maximum of the membership functions in case of parallel connection and a minimum of the membership functions in case of series connection.**

Keywords: Fuzzy reliability, failure data, fuzzy possibility, membership function.

1 INTRODUCTION

The system is comprised of one or more subsystems, all of which are interconnected and share components to accomplish their respective duties. A system can be anything from a single machine to a network made up of the same components that are currently considered to be components. Regarding presentation options, the system is entirely flexible. This greatly depends on the context.

Reliability refers to the extent to which actual system performance matches expectations. This relationship is accounted for by stochastic system model assumptions, which provide a metric for system reliability that is based on component reliability. In order to establish the overall system's reliability, it is necessary to characterize and analyze the properties of each component. A new method for possibilities uncertainty analysis of a simple structure system's reliability has been demonstrated. In this context, the primary objective of this paper is to present a methodology to determine the fuzzy reliability of a simple structure system and investigate the block diagram method, as well as determine the fuzzy rule base to solve a case study.

Sharma and Mukesh K, respectively, demonstrated If a system's behavior can be adequately specified within the framework of probability measure, then its reliability can be defined as the probability that it effectively performs its designated function over a specified time period. In practice, however, system parameters are typically imprecise (fuzzy) due to missing or unavailable information, and the probabilistic approach to traditional reliability analysis is insufficient to account for such inherent uncertainties. To achieve this objective, fuzzy reliability, a novel concept introduced and formalized within the framework of probability theory, is employed.[1]

Qimi Jiang and Chun-Hsien Chen presented a computational model of fuzzy dependability with a focus on the application of random general stress-fuzzy general strength to the resolution of engineering problems. This computational model is founded on a mathematical transition that enables the calculation of fuzzy probability using the computational method of conventional probability. On the basis of this computational model, a numerical technique is provided for calculating the imprecise dependability of mechanical elements, sensors, and so on. This paves the way for the analysis of the dependability of systems composed of components whose dependability is uncertain. The case study validates the efficacy of the method by evaluating the fuzzy reliability analysis of a sensor type used in railway systems. The computational results confirm the algorithm's compatibility with engineering practice.[2]

Fabio Biondini, Franco Bontempi, and Pier Giorgio Malerba developed a very general method for determining the safety of reinforced and prestressed concrete structures. Numerous sources of ambiguity are well-known to influence the exact values of the parameters that regulate the geometrical and mechanical characteristics of such structures. Such characteristics cannot be regarded as fixed parameters in the real world. In the current study, all of these unknowns are modeled using a fuzzy criterion, in which the model is not specified by a singular value but by a range of possible values. Different serviceability and ultimate limit states are used to characterize the reliability issue at the load level. The membership function of the safety factor is computed by solving an associated anti-optimization problem for each limit state's critical interval. The overall strategy of this solution procedure is guided by a genetic algorithm that generates random values for the parameters of the material and geometric nonlinear structure assessments.[3]

Reliability has emerged as a key factor in the design and operation of today's enormous, complex, and expensive mechatronics systems. Modern mechatronics systems cannot operate without the durability and dependability of their constituent parts. However, failure physics modeling and familiarity with probability and statistics are required for the field of reliability theory. Therefore, mathematical dependability models are indispensable to this scientific discipline. Possibly the most prevalent method employed in reliability analysis today is one that evaluates the system as a whole by evaluating its individual components.

1.1 Structure of Paper

The structure of the paper is as follows: The first section presented the introduction which includes the problem statement and objective of the paper along with the literature review. In section 2, a synopsis of the Fuzzy Reliability Block Diagram (FRBD) is presented. The Fuzzy Rule Base (FRB) is presented in section 3. Section four presents the conclusion.

2 FUZZY RELIABILITY BLOCK DIAGRAM (FRBD)

There is always the risk of anything going wrong, whether it involves a technique, a component, a piece of equipment, a system, or a person. A deterministic, oversimplified, and utopian viewpoint allows for 100% reliability. When it comes to functioning in the real world, none of us is perfect. Nothing is impervious to the wear and tear that time and circumstance inevitably inflict. Predicting and avoiding failures is the focus of reliability engineering. In order to evaluate reliability issues, it is important to know the why, how, and how often failures occur, as well as the cost of failure in terms of money, effort, and goodwill. [4]

All problems with system dependability can be traced back to specific tangible defects, which can then be corrected. In reality, almost all prospective failures are inadequately understood, making failure prediction a probabilistic challenge in reliability analysis.

2.1 Series Connection Structure

Any structure or system in which some subset of *n* components is required for proper operation is said to be "series-dependent." All of the pieces in a series system must be operational for the system as a whole to be in a condition where failure is impossible. In a series-configured system, if any part of the system fails, the whole thing will go down with it.[5]

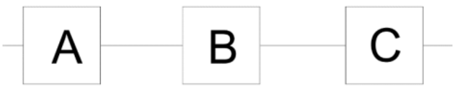

Figure 1: Reliability block diagram for series systems. (WILEY, 2021)

Table 1: Fuzzy set representation series structure data.

μ_{sys}	y	A	B	C
	$\mu_{sys}(y)$	0.9	0.8	0.75

Element (A, B, and C) are connected in series.

$$\mu_{sys} = min\ (\mu_A;\ \mu_B;\ \mu_C);\ \mu_{sys} = 0.75 \quad (1)$$

The Fuzzy Reliability Block Diagram of series connection works when all the sets of the universe or system work which comes from the minimum possibility of membership function in the set or between the sets because the Fuzzy Fault Tree Analysis of series connection comes from the maximum possibility of membership function (OR-gate) in the set or between the sets.

2.2 Parallel Connection Structure

A parallel structure or system is one in which the system is considered to be operational if any one of its *n* components is doing its job.[5]

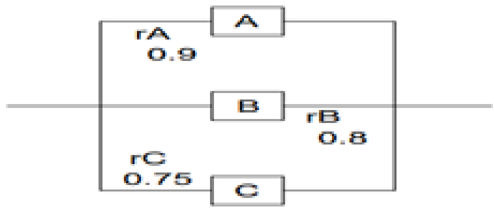

Figure 2: Reliability block diagram for parallel systems. (WILEY, 2021)

In order to provide a failsafe mechanism, it is usual practice to use parallel systems. The Fuzzy Reliability Block Diagram of parallel connection works when one of the sets of the universe or system is working which comes from the maximum possibility of membership function in the set or between the sets because the Fuzzy Fault Tree Analysis of parallel connection it comes from the minimum possibility of membership function (AND-gate) in the set or between the sets.

Table 2: Fuzzy set representation of parallel structure data.

μ_{sys}	y	A	B	C
	$\mu_{sys}(y)$	0.9	0.8	0.75

$$\mu_{sys} = max\ (\mu_A;\ \mu_B;\ \mu_C);\ \mu_{sys} = 0.9 \quad (2)$$

2.3 Combined Connection Structure

The term "redundancy" is used to describe the practice of replicating essential parts or processes in order to boost system reliability (as with a backup or fail-safe) or performance (as with GNSS receivers or multi-threaded computer processing). [6]

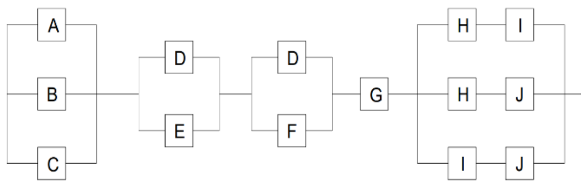

Figure 3: Reliability block diagram study case example.[7]

Table 3: Fuzzy set representation data of study case example.

μ_{sys}	y	A	B	C	D	E
	$\mu_{sys}(y)$	0.9	0.8	0.75	0.85	0.65

μ_{sys}	y	F	G	H	I	J
	$\mu_{sys}(y)$	0.95	0.55	0.7	0.8	0.6

Element (A, B, and C) are connected in Parallel and they become a block 1.

μ_1= max (max (μ_A, μ_B), μ_C);

Element (D and E) are connected in parallel and they become a block 2.

μ_2= max (μ_D, μ_E);

Element (D and F) are connected in parallel and they become block 3.

μ_3= max (μ_D, μ_F);

There is a series connection between block (1,2, and 3) and element G and they become block 4.

μ_4= min (min (μ_1, μ_2), min (μ_3, μ_G));

Element (H and I) are connected in series and they become block 5.

μ_5= min (μ_H, μ_I);

Elements (H and J) are connected in series and they become block 6.

μ_6= min (μ_H, μ_J);

Element (I and J) are connected in series and they become block 7.

μ_7= min (μ_I, μ_J);

There is a parallel connection between blocks (5, 6, and 7) and they become block 8.

μ_8= max (max (μ_5, μ_6), μ_7);

The fuzzy reliability of the system will be a series of connections between blocks (4 and 8).

μ_{sys}= min (μ_4, μ_8)

μ_{sys} = 0.5500

3 Fuzzy Rule-base

In 1965, L. A. Zadeh developed the concept of fuzzy sets for the first time. The objective was to provide a line of reasoning that may be useful in solving problems that have proven difficult to solve in the past. The main innovation was a little tweak to the traditional mathematical concept of sets. This simultaneously defined a membership function and a membership degree, resulting in a blurring of previously separate categories.[8]

Four of the most important components of a Fuzzy rule-based inference system are shown in Figure 4.

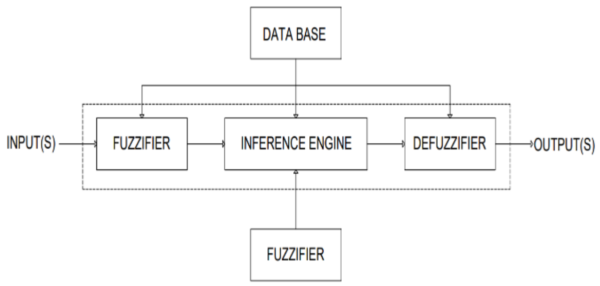

Figure 4: Fuzzy inference system architecture.[8]

Both the input and the output variables are characterized by means of fuzzy sets in the Fuzzifier. At this point, you may substitute metaphorical language and interval numbers for straightforward, simple values.[8]

All of the available knowledge from subject matter experts is collected in the Rule Base and organized according to the identified problem. In order to create the fuzzy rules, we use the conditional expression "IF...(antecedent(s)) ... THEN... (consequence(s)) to build and connect the variables for input and output. Inside the Inference Engine, the rules are activated.[9]

The strength of the current depends on how strongly the independent variables that were utilized to generate the Fuzzy output set are connected to one another. Fuzzy inference comes in many forms, including the Sugeno type, the Mamdani type, and the Tsukamoto type.[10]

One of the most common kinds of inference system, the Mamdani type is used to convey expert knowledge in light of historical data. Here, the conclusion is shown by way of an example. Let's go through the steps of explaining a simple system that has two input variables (x_1 and x_2) and a single output variable (y). The distribution of the variables within their respective classes is shown in Figure 5.[11]

In the situation where x_1 = a and x_2 = b, there are two rules that might be triggered:

$$R_1: IF\ X_1 = A_2\ AND\ X_2 = B_3\ THEN\ Y = C_3 \quad (3)$$

$$R_2: IF\ X_1 = A_3\ AND\ X_2 = B_3\ THEN\ Y = C_2 \quad (4)$$

According to the first rule, the red line represents (a) member's second degree in A_2, whereas the green line represents (b)'s second degree in B_3. The conditions are all interconnected in an AND fashion. Hence, the rule's effectiveness is equal to the lowest membership value. As a result, the red color denotes a C_3 activation state.[10]

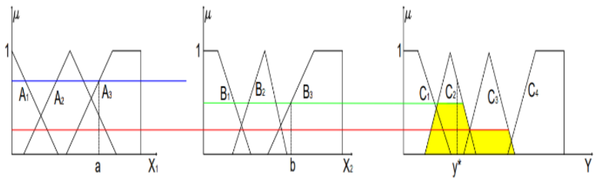

Figure 5: The Mamdani deduction or conclusion.[8]

$R_1: IF\ X_1 = A_2\ AND\ X_2 = B_3\ THEN\ Y = C_3 R_2: IF\ X_1 = A_3\ AND\ X_2 = B_3\ THEN\ Y = C_2$On the other side, the second rule has also been brought into play. The levels of membership are represented by lines that are blue (in the case of A_3) and green (for B_3) respectively. The AND connection may be thought of as reducing their firing power to their minimum, as seen by the green level in the second rule. It is going to be the level (for C_2). In conclusion, the response to the active rules is the conjunction of all the outcomes (the shaded yellow area in Figure 5).[11]

In order to make a more objective evaluation of the resulting Fuzzy set, the next step is to transform it into a form that is more amenable to such an analysis. Specifically, this is a step that the Defuzzifier does before the translation is complete. The illustrative Figure 5 demonstrates the complexities of the result and the resulting perplexity. Therefore, defuzzification is crucial. Hence, a result (y * in Figure 5) that is easier to interpret is attainable. There are a number of different approaches to defuzzification, but no overarching paradigm has been offered for choosing the most effective one. The best solution is the one that is created specifically for the problem at hand.[12]

A fuzzy inference system might be useful if the problem being investigated is shrouded in uncertainty, there is a dearth of information, there are too many outside influences, or the connection between the two is hard to identify.[12]

3.1 Case Study Example for Fuzzy Rule Base (FRB)

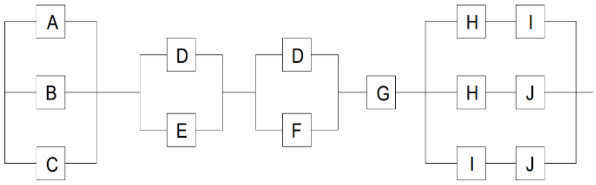

Figure 6: Fuzzy rule base study case example.[7]

Table 3: Fuzzy rule base representation of study case example data.

μ_{sys}	y	A	B	C	D	E
	$\mu_{sys}(y)$	0.9	0.8	0.75	0.85	0.65

μ_{sys}	y	F	G	H	I	J
	$\mu_{sys}(y)$	0.95	0.55	0.7	0.8	0.6

Type	Num. Membership Function	Range
Mamdani	1	[0 1]

Membership Function type	Mf-value
S-shape mf	[0.25 1]

Num. Input	Num. Output	Num. Rules	Defuzzification
18	9	9	Centroid

AND-gate	OR-gate	Implication	Aggregation
min	max	min	max

$$R_1: IF\ (A = mf(\mu_A))\ OR\ (B = mf(\mu_B))\ OR\ (C = mf(\mu_C))\ THEN\ (\mu_1 = mf(\mu_1))$$

$$R_2: IF\ (D = mf(\mu_D))\ OR\ (E = mf(\mu_E))\ THEN\ (\mu_2 = mf(\mu_2))$$

$$R_3: IF\ (D = mf(\mu_D))\ OR\ (F = mf(\mu_F))\ THEN\ (\mu_3 = mf(\mu_3))$$

$$R_4: IF\ (G = mf(\mu_G))\ AND\ (\mu_1 = mf(\mu_1))\ AND\ (\mu_2 = mf(\mu_2))\ AND\ (\mu_3 = mf(\mu_3))\ THEN\ (\mu_4 = mf(\mu_4))$$

$$R_5: IF\ (H = mf(\mu_H))\ AND\ (I = mf(\mu_I))\ THEN\ (\mu_5 = mf(\mu_5))$$

$$R_6: IF\ (H = mf(\mu_H))\ AND\ (J = mf(\mu_J))\ THEN\ (\mu_6 = mf(\mu_6))$$

$$R_7: IF\ (I = mf(\mu_I))\ AND\ (J = mf(\mu_J))\ THEN\ (\mu_7 = mf(\mu_7))$$

$$R_8: IF\ (\mu_5 = mf(\mu_5))\ OR\ (\mu_6 = mf(\mu_6))\ OR\ (\mu_7 = mf(\mu_7))\ THEN\ (\mu_8 = mf(\mu_8))$$

$$R_9: IF\ (\mu_4 = mf(\mu_4)\ AND\ (\mu_8 = mf(\mu_8))\ THEN\ \mu_{System} = mf(\mu_{System})$$

$$\mu_{System} = 0.766$$

[Rules]

1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0, 1 0 0 0 0 0 0 0 0 : 2
0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0, 0 1 0 0 0 0 0 0 0 : 2
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0, 0 0 1 0 0 0 0 0 0 : 2
0 0 0 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0, 0 0 0 1 0 0 0 0 0 : 1
0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0, 0 0 0 0 1 0 0 0 0 : 1
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0, 0 0 0 0 0 1 0 0 0 : 1
0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0, 0 0 0 0 0 0 1 0 0 : 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0, 0 0 0 0 0 0 0 1 0 : 2
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1, 0 0 0 0 0 0 0 0 1 : 1

4 CONCLUSION

The following conclusions can be deduced from the results of analysis:

1. The conventional reliability of a system depends on the probability of the system functioning, whereas the fuzzy reliability of a system depends on the possibility that the system is functioning.

2. In the Fuzzy Reliability Block Diagram (FRBD), the maximum of the membership function is represented by an OR gate connection and is applied for parallel connection whereas the minimum of the membership function is represented by an AND gate connection and is applied for series connection.

3. Calculations are required to determine the criteria of the classic reliability of the system in order to determine which sets can provide the best results, whereas in fuzzy reliability the criteria are the connection structure, and the concept of maximum and minimum is used to determine the criteria of fuzzy reliability.

4. The element with the most significant role in this study is element *G* for the Fuzzy Reliability Block Diagram (FRBD), because element *G* is connected in series with the other elements. This means that if element G's reliability is zero, the entire system's

Reliability is also zero.

5. Fuzzy Reliability Block Diagram (FRBD) can be utilized during troubleshooting when maintenance professionals rely on their generalized experiences. This thinking should be investigated using fuzzy set theory (also known as possibility theory) because these experiences can be interpreted through linguistic variables.

6. In the Fuzzy Rule Base the Mamdani inference system, the S-shape function as a membership function, and the centroid method for defuzzification has been determined, the defuzzifier value represents the fuzzy reliability of the system.

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